CoSine

VOLUME 2
ISSUE 2

Math and Society

A magazine by the Mathematics Society of The Sanskriti School

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Message from the Author

00

‘But Ma’am, what’s the point? What use will we have for this knowledge in real life?’

Every school student has had this concern at some point in their lives. It can often be difficult to imagine the applications of high school mathematics. However, upon closer inspection, it becomes clear that we use mathematics in every aspect of our lives. This is true now, it was true in the past and it will certainly continue to be true in the future.

This edition of CoSine seeks to explore the role that mathematics has played in history, from the ancient mathematicians and the earliest examples of computation to the role of mathematics in the technology of the future. We even look at some of the everyday applications of maths that people might not be aware of, including basketball and online shopping!

At CoSine, we celebrate the role of mathematics in the ordinary and the extraordinary and we seek to foster enthusiasm in other young people for exploring the many uses and applications of the subject. Each of these articles has been written by an inspired young mathematician with a natural curiosity for mathematics.

We hope that the topics we explore in this issue will raise questions for you too, and perhaps prompt you as readers to look out for mathematics in the processes you see shaping the future, as well as those you witness every day.

Addea Gupta

Abstract

Regardless of their reasonable significance, the associations among technology and mathematics have not gotten a lot of academic attention. This article starts by laying out how the technology-mathematics relationship has developed, from the utilization of straightforward helper mémoires for checking and number juggling, by means of the utilization of mathematics in weaving, building and differentexchanges, and the acquaintance of analytics with take care of mechanical issues, to the cutting edge utilization of PCs to take care of both innovative and numerical issues.

There is an extensive amount of information on the connection between technology and maths, yet up 'til now very little has been composed on that among technology and mathematics. Nevertheless, they are firmly associated in a few ways. Modern technology would be inconceivable without mathematics.

The relationship is equal, since mathematics likewise needs technology. Today, mathematicians use PCs for computations, yet additionally for various different undertakings, including the quest for evidences, approvals, and counter-models while likewise being utilized to reproduce simulations of supertasks.

Turing machine

A Turing machine is a mathematical model of computation that characterizes an abstract machine which controls images on a portion of tape as indicated by a table of rules. Despite the model's straightforwardness, given any computer calculation, a Turing machine fit for
reproducing that calculation's rationale can be built.

A Turing machine is a general case of a central processing unit (CPU) that controls all information, with the authoritative machine utilizing successive memory to store information. All the more explicitly, it is a machine competent of enumerating some subjective subset of substantial strings of an alphabet; these strings are a piece of a recursively enumerable set. A Turing machine has a tape of endless length on which it can perform peruse and compose tasks.

The Turing machine is fit for handling an unrestricted punctuation, which further infers that it is prepared to powerfully assess first-request rationale in a vast number of ways. This is broadly shown through lambda analytics.

A Turing machine that can recreate some other Turing machine is called a universal Turing machine (UTM, or essentially a widespread machine). An all the more numerically situated definition with a comparable "general" nature was presented by Alonzo Church, whose chip away at lambda math entwined with Turing's in a proper hypothesis of computation known as the Church–Turing postulation. The proposal expresses that Turing machines to be sure catch the casual idea of effective methods in logic and mathematics, and give an exact meaning of an algorithm or "mechanical system". Contemplating their abstract properties yields numerous bits of knowledge into computer science and complexity hypothesis.

Article 1: Math and Technology

Avyukt Sachdeva

01

Mathematical Theory Behind the Turing Machine

With this encoding of activity tables as strings it gets conceivable on a fundamental level for Turing machines to address inquiries concerning the conduct of other Turing machines. The majority of these inquiries, nonetheless, are undecidable, implying that the capacity being referred to can't be determined precisely. For example, the issue of deciding if a discretionary Turing machine will stop on a specific information, or on all data sources, known as the Halting issue, was demonstrated to be, by and large, undecidable in Turing's unique paper. Rice's hypothesis shows that any non-insignificant inquiry concerning the yield of a Turing machine is undecidable.

A widespread Turing machine can figure any recursive capacity, choose any recursive language, and acknowledge any recursively enumerable language. As indicated by the Church–Turing proposition, the issues feasible by a widespread Turing machine are actually those issues resolvable by a calculation or a successful strategy for calculation, for any sensible meaning of those terms. Thus, an all inclusive Turing machine fills in as a standard against which to analyze computational frameworks, and a framework that can mimic a general Turing machine is called Turing complete.

A dynamic rendition of the widespread Turing machine is the all inclusive capacity, a processable capacity which can be utilized to ascertain some other calculable capacity. The UTM hypothesis demonstrates the presence of such a capacity.

Math dependence on Technology

It is dubious whether any mathematician has ever gone through a restless evening stressing that her notes might in some way have been transformed by unknown forces, supplanting a right verification by a mistaken one. The more typical stress alludes to making a mistake on their own. There is abundant verifiable proof that distributed work, even by profoundly regarded mathematicians, now and again contains genuine mistakes. An example of such as when Einstein due to a calculation error proved that the universe is stagnant instead of it being expanding. This creates an interdependent relationship between maths and technology in which new achievements in mathematics create better technology which can be used by mathematicians to create theories faster and with more precision.

Mathematics is a subject so old that its origin is hard to trace. From tally marks on cave walls, to geometric patterns on ochre rocks, maths has been everywhere in all of human existence.

Although the earliest solid evidence of written maths comes from the ancient Sumerians from Mesopotamia, who developed a complex system of metrology in 3000 BC. Most of the ancient mathematical texts discovered were from Mesopotamia and Egypt, like the Plimpton 322, the Rhind Mathematical Papyrus, and the Moscow Mathematical Papyrus. All of these texts had one thing in common, that all of them talked about Pythagorean triplets. The Plimpton 322 is a Babylonian clay tablet, believed to have been written about 1800 BC. This has a table of 15 rows with 4 columns. It lists 2 out of the 3 numbers, which now are known to form Pythagorean triples, i.e. the positive numbers that satisfy the equation a 2 +b 2 =c 2 . The Rhind Mathematical Papyrus is an Egyptian manuscript, which dates to around 1550 BC. The Moscow Mathematical Papyrus is an ancient Egyptian mathematical manuscript, estimated to be written around 1850 BC. Both of these Egyptian manuscripts consist of problems, regarding triangles, which hint at the discovery of the relation between the sides of a right-angled triangle.

The fact that all of these most ancient mathematical papers, from different geographical areas and somewhat different time periods, hint at the same theory of Pythagorean triplets, is intriguing. It can be deduced from this that the Pythagoras theorem is one of the most ancient and widespread mathematical theory. It shows that this theorem had been developing since long before Pythagoras came along.

Gunika Singh
9F

ARTICLE 2: Ancient Math

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Article 3: The Mechanism of Online Shopping:Credit Cards and Cryptography

Keertana Kartik, XII C

While shopping online, math is possibly one of the furthest things from your mind. However, the RSA algorithm is what protects your credit card information from being stolen when you punch it in to buy shoes. It uses two different keys (known as asymmetric keys). There is a public key, to encode the message (made of two numbers). Anybody can use the public key to encode their information. Once encoded, they cannot decode it. There is also a completely different private key, with the seller/shop. They need this number to decode the information.
What makes the RSA system so efficient is the fact that it uses the product of two primes. When you only have a number that is a product of two prime numbers (eg: 1,70,117), working backwards from that point to get those two numbers is quite difficult. Generally, the RSA uses numbers with over 300 digits, which makes it exceedingly difficult to obtain these two numbers.

To demonstrate how the RSA works; we require two public keys -K and E- and a private key - D.

- Firstly, you choose two large prime numbers (P=3 and Q=11).
- K=PQ=33
- Now, subtract 1 from each prime number and multiply them again
  F=(P-1)(Q-1)=20
- E, the second public key, must picked in a way, such that it is a prime number that does not divide into F and E < F, {E=13}
- D, the private key, is worked out by the following formula-
  D = [(Z . F) + 1] / E
  Here, the value of Z must be adjusted such that D is obtained as a whole number. Thus if we take Z=11, we get D=17.
  Thus, the values involved in the process are-
  - K and E- used to encode the credit card number, everyone can know these. They are calculated using arbitrary prime numbers
  - D - used to decode information, only the receiver knows this. It is calculated using E and K
  - A- the credit card number; and C- the encoded number

The process involves modular arithmetic, which is simply considering the positive remainder as your answer (instead of the quotient) when you divide two numbers.
So if a = bq + r, then (a MOD b) = r

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The Process:

Whenever a customer wants to purchase an item/ sends their credit card information, the computer goes through this entire process, determines K and E and sends them to the customers computer, which encodes the information.

If the credit card number(A)=27

Then it is encoded using K and E by -

$C = (A^E) \text{ MOD } K$

$\Rightarrow C = (27^{13}) \text{MOD } 33$

$\Rightarrow C = (40,52,55,51,53,01,89,76,267) \text{ MOD } 33$

$\Rightarrow C = 15$

The shop decodes the number using D and K by-

$A = (C^D) \text{ MOD } K$

$\Rightarrow A = (15^{17}) \text{ MOD } 33$

$\Rightarrow A= (98.526,125,335,693,359,375) \text{ MOD } 33$

$\Rightarrow A=27$

The reason the RSA is so popular is due to its elegance and simplicity. It was created in 1977 by Ron Rivest, Adi Shamir, and Leonard Adleman.The fundamental logic involved is the difficulty in factorizing K (to get P and Q). Nowadays, for important transactions Kis taken to be of the order of at least 10308 which would take all the computers in the world, put together, longer than the age of the universe to factorize N. As long as we do not come up with extremely efficient and faster ways of factorization, the RSA Code is here to stay in all its elegance, simplicity and beauty.

There are a few things in the world that we usually just take for granted that are going to be true no matter what.You know,tomorrow's cofefe of Trump will be even weirder than today's,my teacher is not going to believe that the random piece of paper in my pencilbox with a reminder to "DO HOMEWORK"" does not have invisible ink on it or the guys believing that the only reason why i have an android phone is because i like android better.But another one of these inevitably true things that we take to be in accordance with reality is that 1 is greater than 0.But the question is,is it? A recently published study concluded that in fact all these years we have been.

Nope that's not true just making sure you're still here.But still can this statement be proved or is it another one of those axioms that some guy sitting in some bathtub thought of at some point of time while he stared at Mars and wrote your horrorscope. I think I messed a few things up but that's not important let' look at the proof.

1 and 0 are real numbers which are an ordered field.

Now an ordered field is a set for which certain properties(field axioms) hold true for any elements x and y.Some properties I am going to use are:

(1). Law of Trichotomy which says that for two numbers a and b,exactly one of the conditions will be true.
a>b
a=B
a<b

(2). For any numbers a and b wherein
a>b
a+c>b+c

(3). For any numbers a and b wherin a>b
ac>bc when c is a nonzero positive number

(4). For any number x and y
-(xy)=(-x)y

We need to do 3 proofs here

Theorem 1:If k,n,m are integers with n>m and k<0,then kn<km.
For k<0 add -k to both sides using (2)
0<-k

So using (3) multiply -k to both sides of
n<m we get
(-k)n<(-k)m using (4)
-(kn)<-(km) add (kn+km) to both sides using (2)

(kn+km)-kn< (kn+km)-km
km<kn

Theorem 2:If n is a nonzero integer then n2=0
Using law of trichotomy
n>0 or n<0

For n>0 multiplying n to both sides using (3)
n2>0

For n<0 multiplying both sides by n using theorem 1
n2>0

In all cases n>0

To prove 1>0
12=1 and 1 is ≠ to 0
Therefore using theorem 2, 1>0

P.S Just so you know:
To prove 1 is ≠0
Take a real no. x ≠0
There is a field axiom which says that for a given field there is a number 1 for which
x.1=x if 1=0
x.0=x
But x ≠0
Hence 1 ≠ 0

Article 4: Should I take My Shot?

By Parth Sarthi

In basketball, every time the offense produces a shot opportunity the player with the ball must decide if the shot is worth taking. In this article, we will see the answer to this question and show that the "lower cutoff" for shot quality depends on the number of shot opportunities left, and contrary to intuition, a larger demands only higher-quality shots be taken. This article is sourced from the findings of research on "The problem of shot selection in basketball" by Brian Skinner.

In basketball, the purpose of the offensive team is to create a high-percentage shot opportunity. Thus, a successful play ends with a player being given the opportunity to take a high-quality shot. At that moment, the player with the ball must make a decision: whether they should take that shot, or pass up the shot and keep possession of the ball and wait for the team to have a higher quality opportunity later on? The answer depends on three factors:

1) the probability that the shot will go in,
2) the distribution of shot quality that the offense is likely to create in the future, and
3) the number of shot opportunities that the offense will have in the future.

Brian Skinner constructs a simple model of the "shoot or pass up the shot" decision to solve for the optimal probability of shooting at each shot opportunity. The model assumes that for each shot opportunity generated by the offense the shot quality is a random variable, independent of all other shot opportunities. At each shot opportunity, is chosen randomly between some minimum shot quality and some maximum . The best numerical definition for is the expected number of points that will be scored by the shot; If all shots are taken to be worth 1 point, then . This allows us to calculate the optimal minimal value of the shot quality such that players shoot if and only if the quality p of the current shot satisfies ,therefore the team's expected score per possession will be maximized.

Note that this "lower cutoff" for shot qual-

ity must depend on the number of plays that are remaining in the possession. That is, their expected score per possession is optimized if they hold on to the ball until an opportunity presents itself for a shot that is essentially certain to go in. On the other hand, if a team has time for only one or two-shot opportunities in a possession, then there is a decent chance that the team will be forced into taking a relatively low-percentage shot. So, intuitively, must increase monotonically with . In the limit (when the current opportunity is the last chance for the team to shoot), we must have : the team should be willing to take even the lowest quality shot. Conversely, in the limit (and, again, in the absence of turnovers),: the team can afford to wait for the "perfect" shot. In the paper, Skinner shows that the solution for at all intermediate values of constitutes a non-trivial sequence that can only be defined recursively. The solution, , "the shooter's sequence" is what we are after.

Skinner comes up with the equation (You can read his proof in the paper linked below). This means that the team should shoot the ball whenever the shot opportunity has a higher quality than the average of what they would get if they held the balland waited for the next position. Realize that The optimal value of is by definition is equal to. Skinner further generalizes the equation to which implies that a team should shoot the ball only when the quality of the current opportunity is greater than the expected value of retaining the ball and getting more shot opportunities. Interestingly, considering the simplicity of the problem statement, this sequence has no exact analytical solution. Its first few terms and its asymptotic limit are as follows:

Note that in the limit where the team has infinite time, their shooting becomes maximally selective (only shots with "perfect" quality should be taken) and maximally efficient (every possession scores points).

This problem of shot selection in basketball should be included to theinteresting and growing research on optimal stopping problems. The question of optimal behavior in sports especially Basketball provides a novel, and highly-applicable playground for mathematics and statistical mechanics.

05

Women in Math
Katherine Johnson

Who is Katherine Johnson? As this question rings out in classrooms, hallways and lecture halls filled with eager math undergraduates it reveals something about the kind of world we live in. Katherine Johnson is the mathematician credited with the calculations that helped put a man on the moon. During her 35 year career at NASA she contributed to several other key projects and also normalised the use of computers in the agency, while completing several complex calculations entirely manually. When we speak of pioneers and geniuses, it's Katherine Johnson we're speaking about. Only we never speak of her.

Katherine Johnson, like countless others before and after her, has been neglected by a society which does not reward the intelligence, bravery and hard work of a woman. In the 1950s USA, women were meant to be wives, mothers and daughters -- never to leave the confines of the home. And those women who did work were to be elevator attendants, secretaries and waitresses, never challenging the intellect of men in superior positions. This was the society in which Katherine Johnson, an african-american woman from Greenbrier county, West Virginia - whose government did not offer public education to communities of color beyond the 8th grade - rose to the top, rubbing shoulders with the rest of the best and brightest in the country.

Unfortunately, our society is much like theirs. In India in 2020, these concerns remain as pertinent as they were in the USA half a century ago. While women are throwing off the shackles of home-making to make their own spaces in the workplace, they face patriarchal oppression, unequal pay and worst of all, sexual harassment. All these are key to understanding India's abysmally low female labour force participation rate -- even among developing countries. Here the experience in the USA and elsewhere can help us: there is no other way to solve these problems than a slow struggle that revolutionises the place of women in our society.

Signs of such struggles are already visible. Last year saw women-led 24/7 sit-in protests spring up across the country. Not only did the materiality of being a woman inform these protests they were home during the workday but the role of women in society was often a key issue being raised. Struggles like Shaheen Bagh today will give way to the Katherine Johnsons of tomorrow. Brave women like her deserve our support. They deserve their place in society. They deserve to be remembered. May it never be asked again -- 'Who is Katherine Johnson?'

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