Practice Test-1

Relations And Functions

Q1 to Q6 carry 1 mark each, Q7 to Q12 carry 4 marks each. **M.M.30**

Q1. Give an example of a relation which is symmetric and reflexive but not transitive.

Q2. Check the injectivity of $f: Z \to Z$ given by $f(x) = x^2$

Q3. If $f = \{(1,3),(2,7),(8,6)\}$ and $g = \{(7,11),(6,0),(3,5)\}$, find gof.

Q4. Let f :R→R be given by $f(x) = (3 - x^3)^{\frac{1}{3}}$, then find $fof(x)$

Q5. If $a * b = \dfrac{a+2}{3b}$, find $(5 * -3) * 1$ where $*$ is a binary operation on R.

Q6. Determine if $*$ associative for the binary operation $*$ defined as $a * b = 2^{ab}$ on Z^+.

Q7. Consider $f: R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible. Find the inverse of f.

Q8. Let R be a relation on the set A of ordered pairs of positive integers defined by $(x, y) R (u, v)$ if and only if $xy = uv$. Show that R is an equivalence relation.

Q9. Let X be a non-empty set. P(X) be its power set. Let '*' be an operation defined on elements P(x) by, Then, prove that * is a binary operation in P. Is * commutative? Is * associative?

Find the identity element in P(X) w. r.t $*$. Find all the invertible elements of P(X).

If $\circ$ is another binary operation defined on P(X) as $A \circ B = A \cup B$, then verify that $\circ$ distributes itself over *.

Q10. Show that the function $f: R \to R$ defined by $f(x) = \dfrac{3x+1}{2}$ is one- one and onto. Hence find the inverse of the function.

Q11. Show that the function $f: A \to B$ defined as $f(x) = \dfrac{3x+4}{5x-7}$, where $A = R - \left\{\dfrac{7}{5}\right\}, B = R - \left\{\dfrac{3}{5}\right\}$ is invertible and hence find f^{-1}.

Q12. Let * be a binary operation on Q, such that a * b = a + b – ab.

(i) Prove that * is commutative and associative.

(ii) Find identity element of * in Q (if it exists).